Web appendix for

Estimating Incentive and Welfare Effects of Non-Stationary Unemployment Benefits

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This appendix contains various analytical derivations for the equations in the main text of the paper. More material related to this paper is available at www.waelde.com/pub. The page contains the matlab code for the numerical solution, some explanations on how to use this code, more background on Volterra equations as used here in (19), the code for estimation including data and an introduction to this estimation code.

A.6 The model

A.6.1 First-order condition for effort

The subjective arrival rate $\mu\left(\phi\left(s\right)\theta,p\left(s\right)\right)$, introduced in the main text just before (5), given the functional form (25) reads $\mu\left(\phi\left(s\right)\theta,p\left(s\right)\right)=\left(\left(1-p\left(s\right)\right)\eta_0+p\left(s\right)\eta_1\right)\left[\phi\left(s\right)\theta\right]^{\alpha}$. The first-order condition (8) then obviously requires to compute[39]

$$\frac{\partial}{\partial\phi\left(s\right)}\mu\left(p\left(s\right),\phi\left(s\right)\theta\right)=\left(\left(1-p\left(s\right)\right)\eta_0+p\left(s\right)\eta_1\right)\alpha\phi\left(s\right)^{\alpha-1}\theta^{\alpha}.$$

Given the functional form (23) for the utility function, we have for (8)

$$\left(\left(1-p\left(s\right)\right)\eta_0+p\left(s\right)\eta_1\right)\alpha\phi\left(s\right)^{\alpha-1}\theta^{\alpha}\left[V\left(w\right)-V\left(b\left(s\right),s\right)\right]=1\Leftrightarrow$$
$$\phi\left(s\right)=\left\{\left(\left(1-p\left(s\right)\right)\eta_0+p\left(s\right)\eta_1\right)\alpha\theta^{\alpha}\left[V\left(w\right)-V\left(b\left(s\right),s\right)\right]\right\}^{1/(1-\alpha)}.$$

With (26), we obtain the expression (A.4) in the main text.

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[39]The assumption of a passive Bayesian learner is implicit in this derivative: The effect of more effort on the evolution of the belief is not taken into account.

A.6.2 The Bellman equation for vacancies

Consider a vacancy for a job of type k. Its present value in t is given by $V(\pi_k(t)) = E_t \int_t^\infty e^{-\rho[\tau-t]}\pi_k(\tau)\,d\tau$ where profits are chosen as state variable. The latter follow $d\pi_k(t) = (\pi_k + \gamma_k)\,dq_k(t)$ with arrival rates ν_k and the initial condition is $\pi_k(t) = -\gamma_k$. In words, when a vacancy is open at t, it creates instantaneous costs of γ_k. With a certain arrival rate ν_k, however, an individual with characteristics k will arrive. Profits then increase from $-\gamma_k$ to π_k where π_k are instantaneous profits made by this firm if the job is filled with an individual k. When we rewrite the present value in a recursive way, we obtain $\rho V(\pi_k(t)) = -\gamma + \nu_k [V(\pi_k) - V(-\gamma_k)]$. Defining $J_{0k} \equiv V(-\gamma_k)$ and $J_k = V(\pi_k)$, we obtain

$$\rho J_{0k} = -\gamma_k + \nu_k [J_k - J_{0k}].$$

The arrival rate ν_k that any vacancy is filled by an unemployed worker of type k is given by the average over all individuals of their rate of finding a job times the number of unemployed of type k divided by the total number of vacancies of type k,

$$\nu_k = \frac{U_k}{V_k}\bar{\mu}_k.$$

The average $\bar{\mu}_k$ is computed as in (11). Noting that there is one θ_k for each group and using the expression from (4), we get

$$\nu_k = \theta_k^{-1}\bar{\mu}_k.$$

The Bellman equation for a vacancy therefore reads

$$\rho J_{0k} = -\gamma_k + \theta_k^{-1}\bar{\mu}_k [J_k - J_{0k}]. \tag{B.1}$$

A.6.3 Nash bargaining

In this section we derive the wage equation (13). In spirit, the derivation in this subsection closely follows the steps in Pissarides (1985) for risk-neutral agents or Lehmann and van der Linden (2007) for risk-averse agents.

Our derivation focuses on industry wide unions, allows for risk-aversion and benefits being determined by a replacement rate. Reflecting the institutional setup in Germany and many other OECD countries, we stipulate that UI payments are given by $b_{UI,k} = \xi_{UI} w_k$ from (3) adapted for a skill group k. This adds another derivative into the wage setting equation.

- Preliminaries

To make this derivation self-contained, we replicate the Bellman equations from the main text. The Bellman equation of an employed worker of group k is given by

$$\rho V(w_k(t)) = u(w_k(t)) + \dot{V}(w_k(t)) + \lambda_k [V(b_k(0), 0) - V(w_k(t))] \tag{B.2}$$

where unemployment benefits $b_k(0)$ at the instant the worker loses the job are now proportional to the wage $w_k(t)$ earned an instant before losing the job,

$$b_k(0) = \xi(0)\, w_k(t). \tag{B.3}$$

The Bellman equation for the unemployed worker from (7) with index k reads

$$\rho V\left(b_k\left(s\right),s\right)=u\left(b_k\left(s\right),\phi_k\left(s\right)\right)+\dot{V}\left(b_k\left(s\right),s\right)+\mu_k\left(\phi_k\left(s\right)\theta_k,p_k\left(s\right)\right)\left[V\left(w_k\left(t\right)\right)-V\left(b_k\left(s\right),s\right)\right]. \tag{B.4}$$

The value of an occupied and vacant job to a firm depends on the gross wage $\frac{w_k(t)}{1-\kappa}$ and is given by

$$\rho J\left(\frac{w_k\left(t\right)}{1-\kappa}\right)=A_k-\frac{w_k\left(t\right)}{1-\kappa}+\dot{J}\left(\frac{w_k\left(t\right)}{1-\kappa}\right)+\lambda_k\left[J_{0k}-J\left(\frac{w_k\left(t\right)}{1-\kappa}\right)\right] \tag{B.5}$$

and the dynamic version of the equation for vacancies (B.1) reads

$$\rho J_{0k}=-\gamma_k+\dot{J}_{0k}+\frac{\bar{\mu}_k}{\theta_k}\left[J\left(\frac{w_k\left(t\right)}{1-\kappa}\right)-J_{0k}\right]. \tag{B.6}$$

Computing *partial* derivatives of $V\left(w_k\left(t\right)\right)$ and $J\left(\frac{w_k(t)}{1-\kappa}\right)$ with respect to $w_k\left(t\right)$ using (B.2) and (B.5) we get

$$\begin{aligned}\frac{\partial V\left(w_k\left(t\right)\right)}{\partial w_k\left(t\right)}&=\frac{1}{\rho+\lambda_k}\left[u_{w_k}\left(w_k\left(t\right)\right)+\lambda_k\frac{\partial V\left(\xi\left(0\right)w_k\left(t\right),0\right)}{\partial w_k\left(t\right)}\right]\\&\equiv\frac{1}{\rho+\lambda_k}\left[u_{w_k}\left(w_k\left(t\right)\right)+\lambda_k V_{w_k(t)}\left(\xi\left(0\right)w_k\left(t\right),0\right)\right] &\text{(B.7)}\\\frac{\partial J\left(\frac{w_k(t)}{1-\kappa}\right)}{\partial w_k\left(t\right)}&=-\frac{1}{\left(1-\kappa\right)\left[\rho+\lambda_k\right]}. &\text{(B.8)}\end{aligned}$$

Compared to a setup with fixed unemployment benefits, the term $\frac{\partial V(\xi(0)w_k(t),0)}{\partial w_k(t)}$ is new - and it is the only new term. In order to compute it, insert $\xi\left(0\right)w_k\left(t\right)$ from (B.3) into (B.4) and evaluate it at $s=0$. This gives

$$\begin{aligned}\rho V\left(\xi\left(0\right)w_k\left(t\right),0\right)&=u\left(\xi\left(0\right)w_k\left(t\right),\phi_k\left(0\right)\right)\\&+\dot{V}\left(\xi\left(0\right)w_k\left(t\right),0\right)+\mu_0\left[V\left(w_k\left(t\right)\right)-V\left(\xi\left(0\right)w_k\left(t\right),0\right)\right]\quad\Leftrightarrow\end{aligned}$$

$$V\left(\xi\left(0\right)w_k\left(t\right),0\right)=\frac{u\left(\xi\left(0\right)w_k\left(t\right),\phi_k\left(0\right)\right)+\dot{V}\left(\xi\left(0\right)w_k\left(t\right),0\right)+\mu_0 V\left(\tilde{w}\left(t\right)\right)}{\rho+\mu_0},$$

where we use

$$\mu_0\equiv\mu_k\left(\phi_k\left(0\right)\theta_k,p_0\right) \tag{B.9}$$

for this appendix and where p_0 is the initial belief introduced just before (27). The new term $\frac{\partial V(\xi(0)w_k(t),0)}{\partial w_k(t)}$ describes the effect of the bargained wage on the value of becoming unemployed. As unemployment benefits are proportional to the wage, this link is taken into account in a bargaining process. We assume, however, that the wage bargained for the current job does not have an impact on the "next" wage, i.e. the wage a worker would earn once he went through an unemployment spell. In order to be clear on the difference between the wage being bargained for the current job and the wage in the next job (and thereby to correctly compute derivatives), we denote the wage for the next job by $\tilde{w}$ in this equation. Hence, the derivative with respect to the wage for the current job becomes

$$\frac{\partial V\left(\xi\left(0\right)w_k\left(t\right),0\right)}{\partial w_k\left(t\right)} \equiv V_{w_k(t)}\left(\xi\left(0\right)w_k\left(t\right),0\right) = \frac{u_w\left(\xi\left(0\right)w_k\left(t\right),\phi_k\left(0\right)\right) + \dot{V}_w\left(\xi\left(0\right)w_k\left(t\right),0\right)}{\rho+\mu_0}. \tag{B.10}$$

Computing now differences in changes, (B.2) and (B.4) imply

$$\begin{aligned}
&\dot{V}\left(w_k\left(t\right)\right) - \dot{V}\left(b_k\left(s\right),s\right)\\
&= \rho V\left(w_k\left(t\right)\right) - u\left(w_k\left(t\right)\right) - \lambda_k\left[V\left(\xi\left(0\right)w_k\left(t\right),0\right) - V\left(w_k\left(t\right)\right)\right]\\
&- \rho V\left(b_k\left(s\right),s\right) + u\left(b_k\left(s\right),\phi_k\left(s\right)\right) + \mu_k\left(\phi_k\left(s\right)\theta_k, p_k\left(s\right)\right)\left[V\left(w_k\left(t\right)\right) - V\left(b_k\left(s\right),s\right)\right]\\
&= \rho\left[V\left(w_k\left(t\right)\right) - V\left(b_k\left(s\right),s\right)\right] + \lambda_k\left[V\left(w_k\left(t\right)\right) - V\left(\xi\left(0\right)w_k\left(t\right),0\right)\right]\\
&+ \mu_k\left(\phi_k\left(s\right)\theta_k, p_k\left(s\right)\right)\left[V\left(w_k\left(t\right)\right) - V\left(b_k\left(s\right),s\right)\right] - u\left(w_k\left(t\right)\right) + u\left(b_k\left(s\right),\phi_k\left(s\right)\right).
\end{aligned}$$

Evaluating this at $s=0$, we get

$$\begin{aligned}
\dot{V}\left(w_k\left(t\right)\right) - \dot{V}\left(b_k\left(0\right),0\right) &= \left\{\rho+\mu_0\right\}\left[V\left(w_k\left(t\right)\right) - V\left(b_k\left(0\right),0\right)\right] \\
&\quad +\lambda_k\left[V\left(w_k\left(t\right)\right) - V\left(\xi\left(0\right)w_k\left(t\right),0\right)\right] - u\left(w_k\left(t\right)\right) + u\left(b_k\left(0\right),\phi_k\left(0\right)\right)
\end{aligned} \tag{B.11}$$

Similarly from (B.5) and (B.6),

$$\rho J_{0k} = -\gamma_k + \dot{J}_{0k} + \frac{\bar{\mu}_k}{\theta_k}\left[J\left(\frac{w_k\left(t\right)}{1-\kappa}\right) - J_{0k}\right],$$

$$\begin{aligned}
\dot{J}\left(\frac{w_k\left(t\right)}{1-\kappa}\right) - \dot{J}_{0k} &= \rho J\left(\frac{w_k\left(t\right)}{1-\kappa}\right) - A_k + \frac{w_k\left(t\right)}{1-\kappa} - \lambda_k\left[J_{0k} - J\left(\frac{w_k\left(t\right)}{1-\kappa}\right)\right]\\
&\quad - \left(\rho J_{0k} + \gamma_k - \frac{\bar{\mu}_k}{\theta_k}\left[J\left(\frac{w_k\left(t\right)}{1-\kappa}\right) - J_{0k}\right]\right)\\
&= \left\{\rho + \lambda_k + \frac{\bar{\mu}_k}{\theta_k}\right\}\left[J\left(\frac{w_k\left(t\right)}{1-\kappa}\right) - J_{0k}\right] - A_k + \frac{w_k\left(t\right)}{1-\kappa} - \gamma_k.
\end{aligned} \tag{B.12}$$

- Nash bargaining

The union and the firms agree on the wage that maximizes the weighted surplus

$$S = \left(V\left(w_k\left(t\right)\right) - V\left(b_k\left(0\right),0\right)\right)^{\beta}\left(J\left(\frac{w_k\left(t\right)}{1-\kappa}\right) - J_{0k}\right)^{1-\beta} L_k\left(t\right). \tag{B.13}$$

With logs, this reads

$$\ln S = \beta\ln\left(V\left(w_k\left(t\right)\right) - V\left(b_k\left(0\right),0\right)\right) + \left(1-\beta\right)\ln\left(J\left(\frac{w_k\left(t\right)}{1-\kappa}\right) - J_{0k}\right) + \ln L_k\left(t\right).$$

The first order condition with respect to $w_k\left(t\right)$ reads

$$\frac{\partial\ln S}{\partial w_k\left(t\right)} = \beta\frac{1}{V\left(w_k\left(t\right)\right) - V\left(b_k\left(0\right),0\right)}\frac{\partial V\left(w_k\left(t\right)\right)}{\partial w_k\left(t\right)} + \left(1-\beta\right)\frac{1}{J\left(\frac{w_k(t)}{1-\kappa}\right) - J_{0k}}\frac{\partial J\left(\frac{w_k(t)}{1-\kappa}\right)}{\partial w_k\left(t\right)} = 0,$$

from which follows that

$$\beta\frac{\partial V\left(w_k\left(t\right)\right)}{\partial w_k\left(t\right)}\left(J\left(\frac{w_k\left(t\right)}{1-\kappa}\right)-J_{0k}\right)=-\left(1-\beta\right)\frac{\partial J\left(\frac{w_k(t)}{1-\kappa}\right)}{\partial w_k\left(t\right)}\left[V\left(w_k\left(t\right)\right)-V\left(b_k\left(0\right),0\right)\right]. \tag{B.14}$$

Note that the steps for Nash bargaining so far are not affected by the assumption on how unemployment benefits are determined - relative or absolute.

Inserting partial derivatives from (B.7) and (B.8) into (B.14), we obtain

$$\beta\left[u_w\left(w_k\left(t\right)\right)+\lambda_k V_w\left(\xi\left(0\right)w_k\left(t\right),0\right)\right]\left(J\left(\frac{w_k\left(t\right)}{1-\kappa}\right)-J_{0k}\right)=\frac{1-\beta}{1-\kappa}\left[V\left(w_k\left(t\right)\right)-V\left(b_k\left(0\right),0\right)\right]. \tag{B.15}$$

- Obtaining an expression without value functions

To solve for $w_k\left(t\right)$ we need to consider the time derivative of (B.15) in which we will substitute out the values of all the states. Differentiating (B.15) with respect to t we get

$$\begin{aligned}&\beta\left[\dot{u}_w\left(w_k\left(t\right)\right)+\lambda_k\dot{V}_w\left(\xi\left(0\right)w_k\left(t\right),0\right)\right]\left(J\left(\frac{w_k\left(t\right)}{1-\kappa}\right)-J_{0k}\right)\\&+\beta\left[u_w\left(w_k\left(t\right)\right)+\lambda_k V_w\left(\xi\left(0\right)w_k\left(t\right),0\right)\right]\left(\dot{J}\left(\frac{w_k\left(t\right)}{1-\kappa}\right)-\dot{J}_{0k}\right)\\&=\frac{1-\beta}{1-\kappa}\left[\dot{V}\left(w_k\left(t\right)\right)-\dot{V}\left(b_k\left(0\right),0\right)\right]\end{aligned} \tag{B.16}$$

Inserting (B.11) and (B.12) into (B.16), we get

$$\begin{aligned}&\beta\left[\dot{u}_w\left(w_k\left(t\right)\right)+\lambda_k\dot{V}_w\left(\xi\left(0\right)w_k\left(t\right),0\right)\right]\left(J\left(\frac{w_k\left(t\right)}{1-\kappa}\right)-J_{0k}\right)\\&+\beta\left[u_w\left(w_k\left(t\right)\right)+\lambda_k V_w\left(\xi\left(0\right)w_k\left(t\right),0\right)\right]\left(\left\{\rho+\lambda_k+q\left(t\right)\right\}\left[J\left(\frac{w_k\left(t\right)}{1-\kappa}\right)-J_{0k}\right]-A_k+\frac{w_k\left(t\right)}{1-\kappa}-\gamma_k\right)\\&=\frac{1-\beta}{1-\kappa}\left[\left\{\rho+\mu_0\right\}\left[V\left(w_k\left(t\right)\right)-V\left(b_k\left(0\right),0\right)\right]+\lambda_k\left[V\left(w_k\left(t\right)\right)-V\left(\xi\left(0\right)w_k\left(t\right),0\right)\right]\right]\\&-\frac{1-\beta}{1-\kappa}\left[u\left(w_k\left(t\right)\right)-u\left(b_k\left(0\right),\phi_k\left(0\right)\right)\right]\end{aligned} \tag{B.17}$$

- Steady state considerations

So far, derivations were perfectly general. We now continue in our derivation of a wage equation by exploiting properties of the steady state. In a model with infinitely living agents, the steady state with sufficiently long history of job transitions implies $b_k\left(0\right)=\xi\left(0\right)w_k$, where w_k is the the steady state wage solution. Furthermore absence of time dimension implies

$$\begin{aligned}&\dot{u}_w\left(w_k\left(t\right)\right)=0,\quad\dot{V}_w\left(\xi\left(0\right)w_k\left(t\right),0\right)=0\quad\text{and}\\&V_w\left(\xi\left(0\right)w_k\left(t\right),0\right)=V_{w_k}\left(\xi\left(0\right)w_k,0\right)=\frac{u_{w_k}\left(b_k\left(0\right),\phi_k\left(0\right)\right)}{\rho+\mu_0}\end{aligned}$$

from (B.10). Thus the steady state counterpart of (B.17) is with (B.10)

$$\beta\left[u_{w_k}(w_k)+\lambda_k\frac{u_{w_k}(b_k(0),\phi_k(0))}{\rho+\mu_0}\right]\left(\left\{\rho+\lambda_k+\frac{\bar{\mu}_k}{\theta_k}\right\}\left[J\left(\frac{w_k}{1-\kappa}\right)-J_{0k}\right]-A_k+\frac{w_k}{1-\kappa}-\gamma_k\right)$$
$$=\frac{1-\beta}{1-\kappa}\{\rho+\lambda_k+\mu_0\}\left[V(w_k)-V(b_k(0),0)\right]-\frac{1-\beta}{1-\kappa}\left[u(w_k)-u(b_k(0),\phi_k(0))\right]. \tag{B.18}$$

The steady state counterpart of (B.15) implies that the difference between $V(w_k)$ and $V(b_k(0),0)$ is

$$V(w_k)-V(b_k(0),0)=\beta\frac{1-\kappa}{1-\beta}\left[u_{w_k}(w_k)+\lambda_k\frac{u_{w_k}(b_k(0),\phi_k(0))}{\rho+\mu_0}\right]\left(J\left(\frac{w_k}{1-\kappa}\right)-J_{0k}\right).$$

Inserting this into (B.18) gives

$$\beta\left[u_{w_k}(w_k)+\lambda_k\frac{u_{w_k}(b_k(0),\phi_k(0))}{\rho+\mu_0}\right]\left(\left\{\rho+\lambda_k+\frac{\bar{\mu}_k}{\theta_k}\right\}\left[J\left(\frac{w_k}{1-\kappa}\right)-J_{0k}\right]-A_k+\frac{w_k}{1-\kappa}-\gamma_k\right)$$
$$=\beta\{\rho+\lambda_k+\mu_0\}\left[u_{w_k}(w_k)+\lambda_k\frac{u_{w_k}(b_k(0),\phi_k(0))}{\rho+\mu_0}\right]\left(J\left(\frac{w_k}{1-\kappa}\right)-J_{0k}\right)$$
$$-\frac{1-\beta}{1-\kappa}\left[u(w_k)-u(b_k(0),\phi_k(0))\right].$$

and rearranging we get

$$\beta\left[u_{w_k}(w_k)+\lambda_k\frac{u_{w_k}(b_k(0),\phi_k(0))}{\rho+\mu_0}\right]\left(J\left(\frac{w_k}{1-\kappa}\right)-J_{0k}\right)\left\{\frac{\bar{\mu}_k}{\theta_k}-\mu_0\right\}$$
$$=\beta\left[u_{w_k}(w_k)+\lambda_k\frac{u_{w_k}(b_k(0),\phi_k(0))}{\rho+\mu_0}\right]\left[A_k-\frac{w_k}{1-\kappa}+\gamma_k\right]-\frac{1-\beta}{1-\kappa}\left[u(w_k)-u(b_k(0),\phi_k(0))\right].$$

Applying the free entry condition $J_{0k}=0$ in the form of $J\left(\frac{w_k}{1-\kappa}\right)=\frac{\gamma_k\theta_k}{\bar{\mu}_k}$ from (12) and substituting this into the above equation, we obtain

$$\beta\left[u_{w_k}(w_k)+\lambda_k\frac{u_{w_k}(b_k(0),\phi_k(0))}{\rho+\mu_0}\right]\frac{\gamma_k\theta_k}{\bar{\mu}_k}\left\{\frac{\bar{\mu}_k}{\theta_k}-\mu_0\right\}$$
$$=\beta\left[u_{w_k}(w_k)+\lambda_k\frac{u_{w_k}(b_k(0),\phi_k(0))}{\rho+\mu_0}\right]\left[A_k-\frac{w_k}{1-\kappa}+\gamma_k\right]-\frac{1-\beta}{1-\kappa}\left[u(w_k)-u(b_k(0),\phi_k(0))\right]$$

$$\Leftrightarrow\quad\beta\left[\gamma_k-\frac{\gamma_k\theta_k}{\bar{\mu}_k}\mu_0\right]=\beta\left[A_k-\frac{w_k}{1-\kappa}+\gamma_k\right]-\frac{1-\beta}{1-\kappa}\frac{u(w_k)-u(b_k(0),\phi_k(0))}{u_{w_k}(w_k)+\frac{\lambda_k}{\rho+\mu_0}u_{w_k}(b_k(0),\phi_k(0))}$$
$$\Leftrightarrow\quad\frac{1-\beta}{1-\kappa}\frac{u(w_k)-u(b_k(0),\phi_k(0))}{u_{w_k}(w_k)+\frac{\lambda_k}{\rho+\mu_0}u_{w_k}(b_k(0),\phi_k(0))}=\beta\left[A_k-\frac{w_k}{1-\kappa}+\frac{\gamma_k\theta_k}{\bar{\mu}_k}\mu_0\right]$$
$$\Leftrightarrow\quad\beta w_k+(1-\beta)\frac{u(w_k)-u(b_k(0),\phi_k(0))}{u_{w_k}(w_k)+\frac{\lambda_k}{\rho+\mu_0}u_{w_k}(b_k(0),\phi_k(0))}=\beta(1-\kappa)\left[A_k+\frac{\gamma_k\theta_k}{\bar{\mu}_k}\mu_0\right]$$

where $b_k(0) = \xi(0) w_k$.

If we reintroduce $\mu_k(\phi_k(0)\theta_k, p_0)$ for μ_0 from (B.9) and furthermore define generalized marginal utility

$$m_{w_k}(w_k, b_k(0), \phi_k(0)) \equiv u_{w_k}(w_k) + \frac{\lambda_k}{\rho + \mu_k(\phi_k(0)\theta_k, p_0)} u_{w_k}(b_k(0), \phi_k(0)), \qquad \text{(B.19)}$$

the wage equation can be more conveniently written as

$$\begin{aligned} &(1-\beta) u(w_k) + \beta m_{w_k}(.) w_k \\ &\quad = (1-\beta) u(b_k(0), \phi_k(0)) + \beta(1-\kappa) m_{w_k}(.) \left[A_k + \gamma_k \frac{\theta_k}{\bar{\mu}_k} \mu_k(\phi_k(0)\theta_k, p_0) \right]. \end{aligned} \qquad \text{(B.20)}$$

Replacing $b_k(0)$ by $b_{UI,k}$ as implied by (2), we obtain the wage equation (13) in the text.

- Numerical version

For the numerical implementation, we use (23) with $b_k(0) = \xi(0) w_k$ from (3) and write

$$\begin{aligned} u_{w_k}(b_k(0), \phi_k(0)) &\equiv \frac{\partial u(b_k(0), \phi_k(0))}{\partial w_k} = \frac{\partial \left(\frac{b_k(0)^{1-\sigma}-1}{1-\sigma} - \phi_k(0) \right)}{\partial w_k} \\ &= \frac{\partial \left(\frac{(\xi(0) w_k)^{1-\sigma}-1}{1-\sigma} - \phi_k(0) \right)}{\partial w_k} = \frac{\partial \frac{(\xi(0) w_k)^{1-\sigma}}{1-\sigma}}{\partial w_k} = \xi(0)^{1-\sigma} w_k^{-\sigma}. \end{aligned}$$

This allows us to express (B.19) as

$$m_{w_k}(.) \equiv w_k^{-\sigma} + \frac{\lambda_k}{\rho + \mu_0} \xi(0)^{1-\sigma} w_k^{-\sigma} = \left(1 + \frac{\lambda_k \xi(0)^{1-\sigma}}{\rho + \mu_0} \right) w_k^{-\sigma}$$

and the wage equation as

$$\frac{1-\beta}{\beta} \left(u(w_k) - u(b_k(0), \phi(0)) \right) + m_{w_k}(.) \left[w_k - (1-\kappa) \left[A_k + \gamma_k \frac{\theta_k}{\bar{\mu}_k} \mu_0 \right] \right] = 0 \quad \Leftrightarrow$$

$$\begin{aligned} &\frac{1-\beta}{\beta} \left(\frac{w_k^{1-\sigma}-1}{1-\sigma} - \frac{b_{UI}^{1-\sigma}-1}{1-\sigma} + \phi_k(0) \right) \\ &\qquad + m_{w_k}(.) \left[w_k - (1-\kappa) \left[A_k + \frac{A_k - w_k/(1-\kappa)}{\rho + \lambda_k} \mu_0 \right] \right] = 0, \end{aligned} \qquad \text{(B.21)}$$

where the last step also used (A.11) written as $\frac{\gamma_k \theta_k}{\bar{\mu}_k} = \frac{A_k - w_k/(1-\kappa)}{\rho + \lambda_k}$ and we used μ_0 from (B.9) as above.

A.6.4 Matching functions and individual arrival rates

Let us now use an alternative assumption to (4), i.e. let us assume $\theta \equiv V/(\Omega U)$ where $\Omega \equiv \int p(s)\phi(s)^{\alpha} f(s)\,ds$ as in the main text. As Poisson processes are additive, the matching rate is given, as before, by $m(N-L,V)=(N-L)\int \mu(s) f(s)\,ds$. Inserting (24) where θ is now $\theta \equiv V/(\Omega U)$, we get

$$\begin{aligned} m(N-L,V) &= (N-L)\int p(s)\left[\phi(s)\theta\right]^{\alpha} f(s)\,ds = \Omega\left[N-L\right]\theta^{\alpha} \\ &= \Omega\left[N-L\right]\left(\frac{V}{\Omega U}\right)^{\alpha} = \Omega\left[N-L\right]\left(\frac{V}{\Omega\left[N-L\right]}\right)^{\alpha} = \left(\Omega\left[N-L\right]\right)^{1-\alpha} V^{\alpha}. \end{aligned}$$

The difference in the aggregate matching function is therefore simply the power $1-\alpha$ now applied also to Ω. This is clearly a qualitative difference. As stressed in fn. 26 in the main text, however, we do not believe that this alternative definition of θ will be of major quantitative importance.

A.7 Equilibrium properties

A.7.1 Deriving (A.5) in block 1

Given optimal effort from (A.4), the Bellman equation for the unemployed (7),

$$\rho V(b(s),s) = \max_{\phi(s)}\left\{u(b(s),\phi(s)) + \frac{dV(b(s),s)}{ds} + \mu(\phi(s)\theta, p(s))\left[V(w)-V(b(s),s)\right]\right\}$$

and the subjective arrival rate $\mu(\phi(s)\theta,p(s)) = \eta(s)\left[\phi(s)\theta\right]^{\alpha}$ from (25), we obtain

$$\rho V(b(s),s) = u(b(s),\phi(s)) + \frac{dV(b(s),s)}{ds} + \eta(s)\phi(s)^{\alpha}\theta^{\alpha}\left[V(w)-V(b(s),s)\right].$$

Inserting the utility function (23) and effort $\phi(s) = \left\{\alpha\eta(s)\theta^{\alpha}\left[V(w)-V(b(s),s)\right]\right\}^{1/(1-\alpha)}$ from (A.4), the Bellman equation reads

$$\begin{aligned} \rho V(b(s),s) = {} & \frac{b(s)^{1-\sigma}-1}{1-\sigma} - \left\{\alpha\eta(s)\theta^{\alpha}\left[V(w)-V(b(s),s)\right]\right\}^{1/(1-\alpha)} + \frac{dV(b(s),s)}{ds} \\ & + \eta(s)\left\{\alpha\eta(s)\theta^{\alpha}\left[V(w)-V(b(s),s)\right]\right\}^{\alpha/(1-\alpha)}\theta^{\alpha}\left[V(w)-V(b(s),s)\right]. \end{aligned}$$

Now write

$$\begin{aligned} & \eta(s)\left\{\alpha\eta(s)\theta^{\alpha}\left[V(w)-V(b(s),s)\right]\right\}^{\alpha/(1-\alpha)}\theta^{\alpha}\left[V(w)-V(b(s),s)\right] \\ & \qquad = \eta(s)^{1/(1-\alpha)}\theta^{\alpha/(1-\alpha)}\alpha^{\alpha/(1-\alpha)}\left[V(w)-V(b(s),s)\right]^{1/(1-\alpha)}. \end{aligned}$$

The Bellman equation can then be written as

$$\begin{aligned} \rho V(b(s),s) = {} & \frac{b(s)^{1-\sigma}-1}{1-\sigma} - \left\{\alpha\eta(s)\theta^{\alpha}\left[V(w)-V(b(s),s)\right]\right\}^{1/(1-\alpha)} + \frac{dV(b(s),s)}{ds} \\ & + \eta(s)^{1/(1-\alpha)}(\alpha\theta)^{\alpha/(1-\alpha)}\left[V(w)-V(b(s),s)\right]^{1/(1-\alpha)}. \end{aligned}$$

Now we use

$$
\begin{aligned}
& -\left\{\alpha\eta\left(s\right)\theta^{\alpha}\right\}^{1/(1-\alpha)}+\eta\left(s\right)^{1/(1-\alpha)}\left(\alpha\theta\right)^{\alpha/(1-\alpha)} \\
= & \ \eta\left(s\right)^{1/(1-\alpha)}\theta^{\alpha/(1-\alpha)}\left[-\alpha^{1/(1-\alpha)}+\alpha^{\alpha/(1-\alpha)}\right]=\eta\left(s\right)^{1/(1-\alpha)}\theta^{\alpha/(1-\alpha)}\alpha^{1/(1-\alpha)}\left[-1+\alpha^{-1}\right] \\
= & \ \frac{1-\alpha}{\alpha}\eta\left(s\right)^{1/(1-\alpha)}\theta^{\alpha/(1-\alpha)}\alpha^{1/(1-\alpha)}=\frac{1-\alpha}{\alpha}\left(\alpha\eta\left(s\right)\theta^{\alpha}\right)^{1/(1-\alpha)}
\end{aligned}
$$

and find

$$
\rho V\left(b\left(s\right),s\right)=\frac{b\left(s\right)^{1-\sigma}-1}{1-\sigma}+\frac{dV\left(b\left(s\right),s\right)}{ds}+\frac{1-\alpha}{\alpha}\left(\alpha\eta\left(s\right)\theta^{\alpha}\right)^{1/(1-\alpha)}\left[V\left(w\right)-V\left(b\left(s\right),s\right)\right]^{1/(1-\alpha)}.
$$

The Bellman equation in differential notation therefore reads

$$
\frac{dV\left(b\left(s\right),s\right)}{ds}=\rho V\left(b\left(s\right),s\right)-\frac{b\left(s\right)^{1-\sigma}-1}{1-\sigma}-\frac{1-\alpha}{\alpha}\left(\alpha\eta\left(s\right)\theta^{\alpha}\right)^{1/(1-\alpha)}\left[V\left(w\right)-V\left(b\left(s\right),s\right)\right]^{1/(1-\alpha)}.
$$

which is (A.5) in the main text.

A.7.2 Deriving an expression for $V\left(b_{UI},0\right)$ for block 1

This derives the expression for $V\left(b_{UI},0\right)$ used in the numerical implementation.

From (A.8) with (A.9), write as

$$
V\left(w\right)=\frac{\frac{w^{1-\sigma}-1}{1-\sigma}+\lambda V\left(b_{UI},0\right)}{\rho+\lambda},
$$

we get

$$
\rho V\left(b_{UA}\right)=\frac{b_{UA}^{1-\sigma}-1}{1-\sigma}+\frac{1-\alpha}{\alpha}\left\{\alpha\eta_0\theta^{\alpha}\right\}^{1/(1-\alpha)}\left[\frac{\frac{w^{1-\sigma}-1}{1-\sigma}+\lambda V\left(b_{UI},0\right)}{\rho+\lambda}-V\left(b_{UA}\right)\right]^{1/(1-\alpha)}.
$$

Solving this for $V\left(b_{UI},0\right)$ yields

$$
\frac{\rho V\left(b_{UA}\right)-\frac{b_{UA}^{1-\sigma}-1}{1-\sigma}}{\frac{1-\alpha}{\alpha}\left\{\alpha\eta_0\theta^{\alpha}\right\}^{1/(1-\alpha)}}=\left[\frac{\frac{w^{1-\sigma}-1}{1-\sigma}+\lambda V\left(b_{UI},0\right)}{\rho+\lambda}-V\left(b_{UA}\right)\right]^{1/(1-\alpha)} \Leftrightarrow
$$

$$
\left(\rho+\lambda\right)\left(\left(\frac{\rho V\left(b_{UA}\right)-\frac{b_{UA}^{1-\sigma}-1}{1-\sigma}}{\frac{1-\alpha}{\alpha}\left\{\alpha\eta_0\theta^{\alpha}\right\}^{1/(1-\alpha)}}\right)^{1-\alpha}+V\left(b_{UA}\right)\right)=\frac{w^{1-\sigma}-1}{1-\sigma}+\lambda V\left(b_{UI},0\right) \Leftrightarrow
$$

$$
V\left(b_{UI},0\right)=\frac{1}{\lambda}\left[\left(\rho+\lambda\right)\left(\left(\frac{\rho V\left(b_{UA}\right)-\frac{b_{UA}^{1-\sigma}-1}{1-\sigma}}{\frac{1-\alpha}{\alpha}\left\{\alpha\eta_0\theta^{\alpha}\right\}^{1/(1-\alpha)}}\right)^{1-\alpha}+V\left(b_{UA}\right)\right)-\frac{w^{1-\sigma}-1}{1-\sigma}\right].
$$

A.8 The reform and economic growth

Figure 8 shows aggregate effects of the reform (simultaneous reduction of UA payments b_{UA} and entitlement length $\bar{s}$) combined with a positive productivity shock.

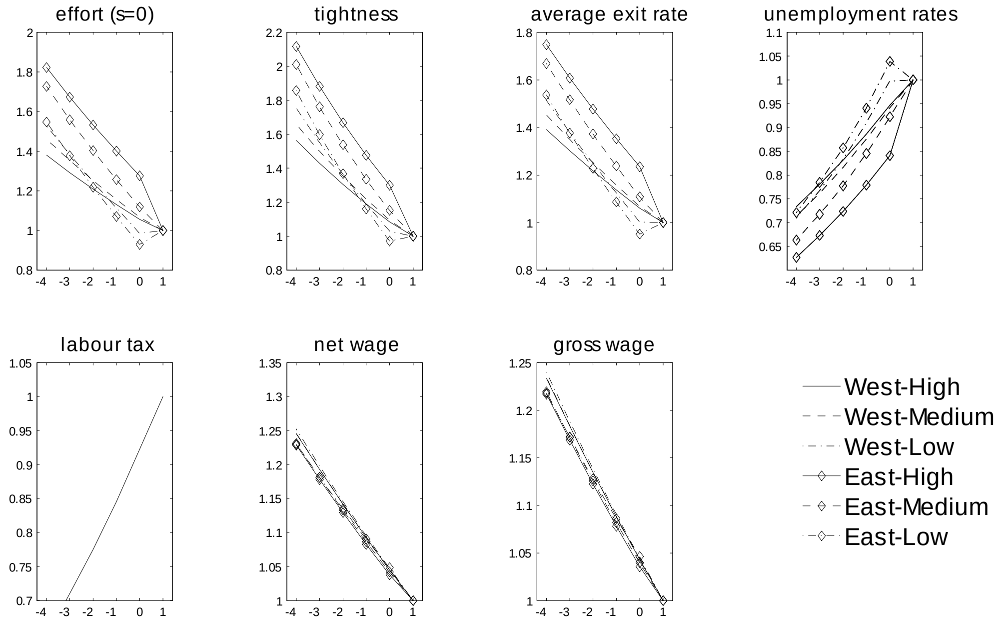

Figure 8 *Aggregate effects of UA payments b_{UA} and entitlement length $\bar{s}$*

Figure 9 shows welfare implications effects of the reform combined with a positive productivity shock. For discussion see towards p. 33 in the main text.

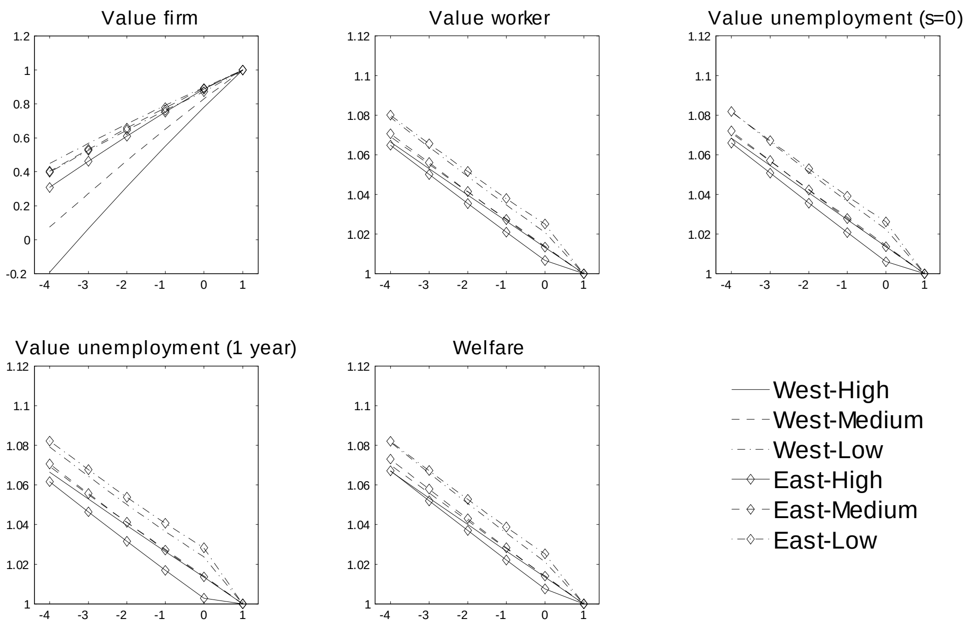

Figure 9 *Aggregate effects of UA payments b_{UA} and entitlement length $\bar{s}$*