This is the complete appendix to

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International competition, growth and welfare

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Appendix 1: The reduced form of the world economy

Equation (12) is found by inserting (8) into $\dot{\eta}/\eta = \dot{n}_m/K_n = \dot{n}_m^A/K_n + \dot{n}_m^B/K_n$. To this end, rearrange (8) to $\frac{\dot{n}_m^i}{K_n} = L^i - n_m^i x_m^i - n_d s^i x_d$ and obtain $\dot{\eta}/\eta = L - n_m^A x_m^A - n_m^B x_m^B - n_d x_d$. Observing that demand for monopolistic varieties is the same independently of their origin (as prices are identical which in turn results from factor price equalization), inserting demand functions (3) yields $\frac{\dot{\eta}}{\eta} = L - n_m \frac{p_m^{-\varepsilon}}{n_m p_m^{1-\varepsilon} + n_d p_d^{1-\varepsilon}} E - n_d \frac{p_d^{-\varepsilon}}{n_m p_m^{1-\varepsilon} + n_d p_d^{1-\varepsilon}} E$ $= L - \frac{n_m p_m^{-\varepsilon} + n_d p_d^{-\varepsilon}}{n_m p_m^{1-\varepsilon} + n_d p_d^{1-\varepsilon}} E = L - \frac{n_m p_m^{1-\varepsilon} + n_d \mu^{-\varepsilon} p_m^{1-\varepsilon}}{n_m p_m^{1-\varepsilon} + n_d p_d^{1-\varepsilon}} \frac{E}{p_m} = L - \frac{n_m + n_d \mu^{-\varepsilon}}{n_m + n_d \mu^{1-\varepsilon}} \alpha \frac{E}{w}$, where the last but one equality used $p_d = \mu p_m$. As $\alpha E w^{-1} = \alpha \delta$ and $\frac{n_m + n_d \mu^{-\varepsilon}}{n_m + n_d \mu^{1-\varepsilon}}$ $= \frac{n_m + n_d + n_d \mu^{-\varepsilon} - n_d}{n_m + n_d + n_d \mu^{1-\varepsilon} - n_d} = \frac{\eta + \mu^{-\varepsilon} - 1}{\eta + \mu^{1-\varepsilon} - 1} = \frac{\eta + \mu^{-\varepsilon} - 1 + \mu^{1-\varepsilon} - \mu^{1-\varepsilon}}{\eta + \mu^{1-\varepsilon} - 1} = 1 + \frac{\mu^{-\varepsilon} - \mu^{1-\varepsilon}}{\eta + \mu^{1-\varepsilon} - 1}$, we obtain (12).

Equation (13) can be obtained by differentiating (7) with respect to time, $\pi_m + \dot{v}_m = r v_m$, and inserting this with the expenditure equation (2) into $\dot{\delta}/\delta = \dot{E}/E - \dot{n}/n - \dot{v}_m/v_m$ $= -\rho - \dot{\eta}/\eta + \pi_m/v_m$, where $\dot{n}/n = \dot{\eta}/\eta$ has been used. The profit ratio can be written as $\frac{\pi_m}{v_m}$

$= (1-\alpha)\dfrac{p_m x_m}{v_m} \quad = (1-\alpha)\dfrac{p_m^{1-\varepsilon}}{n_m p_m^{1-\varepsilon} + n_d p_d^{1-\varepsilon}}\dfrac{E}{v_m} \quad = \dfrac{(1-\alpha)(n_m + n_d)}{n_m + n_d \mu^{1-\varepsilon}}\delta$

$= \dfrac{(1-\alpha)(n_m + n_d)}{n_m + n_d + n_d\mu^{1-\varepsilon} - n_d}\delta \quad = \dfrac{(1-\alpha)\eta}{\eta + \mu^{1-\varepsilon} - 1}\delta$ and inserting gives $\dot{\delta}/\delta$

$= -\rho - L + \alpha\left(1 + \dfrac{\mu^{-\varepsilon}(1-\mu)}{\eta + \mu^{1-\varepsilon} - 1}\right)\delta + \dfrac{(1-\alpha)\eta}{\eta + \mu^{1-\varepsilon} - 1}\delta = \left(\alpha + \dfrac{\alpha\mu^{-\varepsilon}(1-\mu) + (1-\alpha)\eta}{\eta + \mu^{1-\varepsilon} - 1}\right)\delta - \rho - L$. As

$\alpha + \dfrac{\alpha\mu^{-\varepsilon}(1-\mu) + (1-\alpha)\eta}{\eta + \mu^{1-\varepsilon} - 1} \quad = \dfrac{\alpha\left(\eta + \mu^{1-\varepsilon} - 1\right) + \alpha\mu^{-\varepsilon} - \alpha\mu^{1-\varepsilon} + (1-\alpha)\eta}{\eta + \mu^{1-\varepsilon} - 1} \quad = \dfrac{-\alpha + \alpha\mu^{-\varepsilon} + \eta}{\eta + \mu^{1-\varepsilon} - 1}$

$= \dfrac{-\alpha\left(1 - \mu^{-\varepsilon}\right) + \eta + \mu^{1-\varepsilon} - 1 - \mu^{1-\varepsilon} + 1}{\eta + \mu^{1-\varepsilon} - 1} \quad = 1 + \dfrac{1 - \mu^{1-\varepsilon} - \alpha\left(1 - \mu^{-\varepsilon}\right)}{\eta + \mu^{1-\varepsilon} - 1}$, we obtain (13).

When $\eta < \underline{\eta}$ in the no-growth trap and therefore $\dot{\eta} = 0$, we obtain

$\dot{\delta}/\delta = -\rho + \pi_m/v_m \quad = \dfrac{(1-\alpha)\eta}{\eta + \mu^{1-\varepsilon} - 1}\delta - \rho.$

Appendix 2: Further derivations

Deriving equation (9)

In autarky, the labor market clearing condition can be written as

$$\frac{\dot{n}}{n}\frac{n}{K_n} = L - n\frac{E}{np}.$$

This equation shows the assumption that knowledge spillovers are general and denoted by K_n.

Replacing $\dot{n}/n$ by the growth rate g yields

$$g = \frac{K_n}{n}\left(L - \frac{E}{p}\right) = \frac{K_n}{n}L - \alpha\frac{E}{vn}, \tag{A.1}$$

where we have used

$$\frac{E}{p} = \alpha\frac{E}{w} = \alpha\frac{E}{vK_n}.$$

From the derivative of the free entry condition, $\pi + \dot{v} = rv$, and using $\dot{v}/v = -g$ and $r=\rho$ by choice numeraire, we obtain

$$-g = \rho - \frac{\pi}{v} = \rho - (1-\alpha)\frac{px}{v} = \rho - (1-\alpha)\frac{\overline{E}}{vn}. \tag{A.2}$$

Adding (A.1) to (A.2) gives

$$0 = \frac{K_n}{n}L - \alpha\frac{\overline{E}}{vn} + \rho - (1-\alpha)\frac{\overline{E}}{vn} = \frac{K_n}{n}L - \frac{\overline{E}}{vn} + \rho$$

$$\Leftrightarrow \frac{\overline{E}}{vn} = \frac{K_n}{n} L + \rho.$$

Reinserting into (A.1), we obtain

$$g = \frac{K_n}{n} L - \alpha\left(\frac{K_n}{n} L + \rho\right) = (1-\alpha)\frac{K_n}{n} L - \alpha\rho.$$

Deriving equation (19)

The derivative of the gain function in (18) is given by

$$G'\left(g^i\right) = \alpha \frac{L^i - g^i}{L^i}\left[-\frac{(-1)L_i}{(L^i - g^i)^2}\right] - \frac{1-\alpha}{\rho}$$

$$= \alpha \frac{1}{L^i - g^i} - \frac{1-\alpha}{\rho} > 0.$$

Rearranging gives

$$\alpha\rho > (1-\alpha)L^i \;\; -(1-\alpha)g^i \Leftrightarrow g^i > \frac{1}{1-\alpha}\left((1-\alpha)L^i - \alpha\rho\right).$$